Fuzzy Sets and Fuzzy Logic

Fuzzy if-then rules

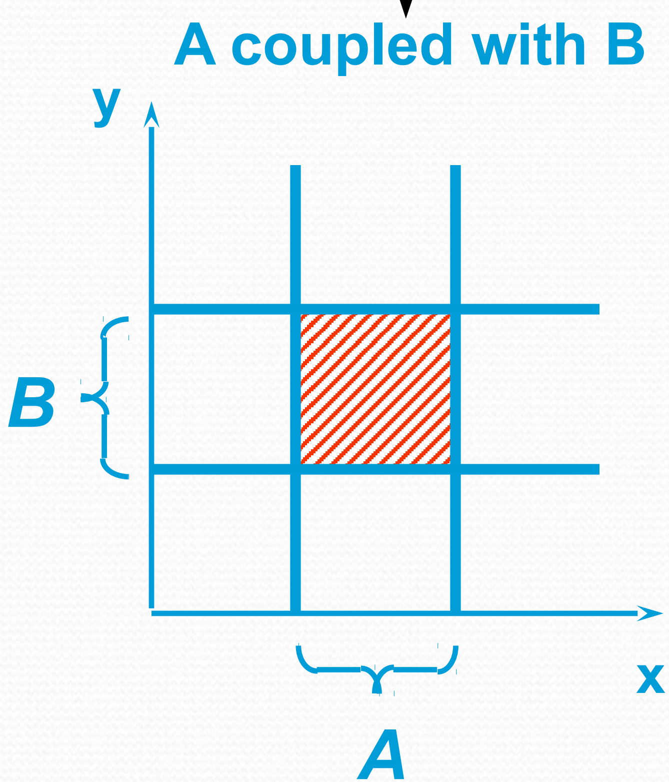
- General format:

  If x is A then y is B

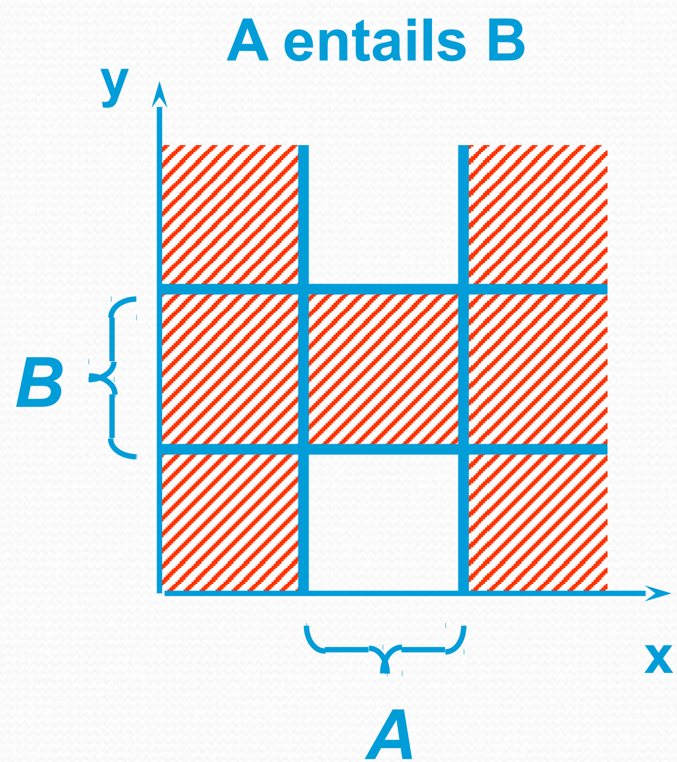
- Examples:
  - If pressure is high, then volume is small
  - If a restaurant is expensive, then order small dishes
  - If a tomato is red, then it is ripe
  - If the speed is high, then apply the brake a little

Interpretation of Implication

Common in Fuzzy logic

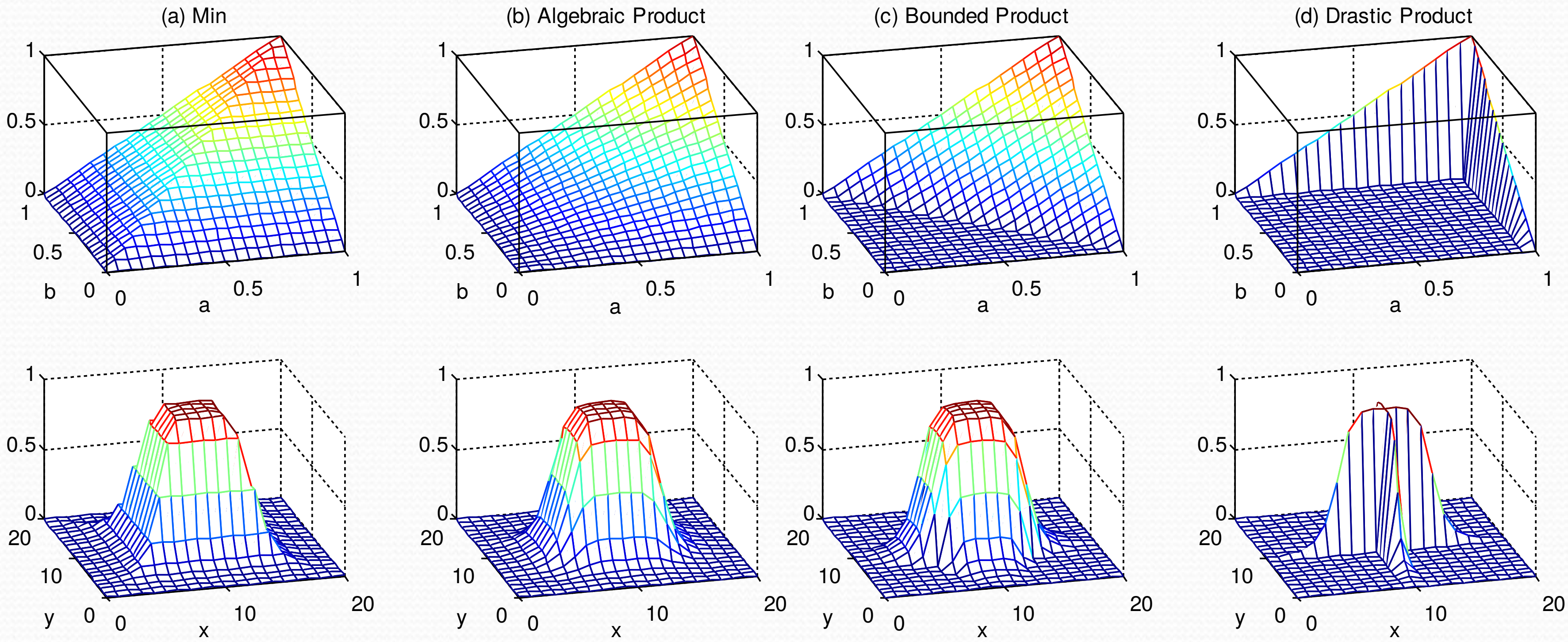


Implication in traditional logic



A coupled with B

Use the T-norm...



A entails B

- Boolean fuzzy implication (based on $\neg A \vee B$)

$$m_R(x, y) = max(1 - m_A(x), m_B(y))$$

- Zadeh's max-min implication (based on $\neg A \vee (A \wedge B)$)

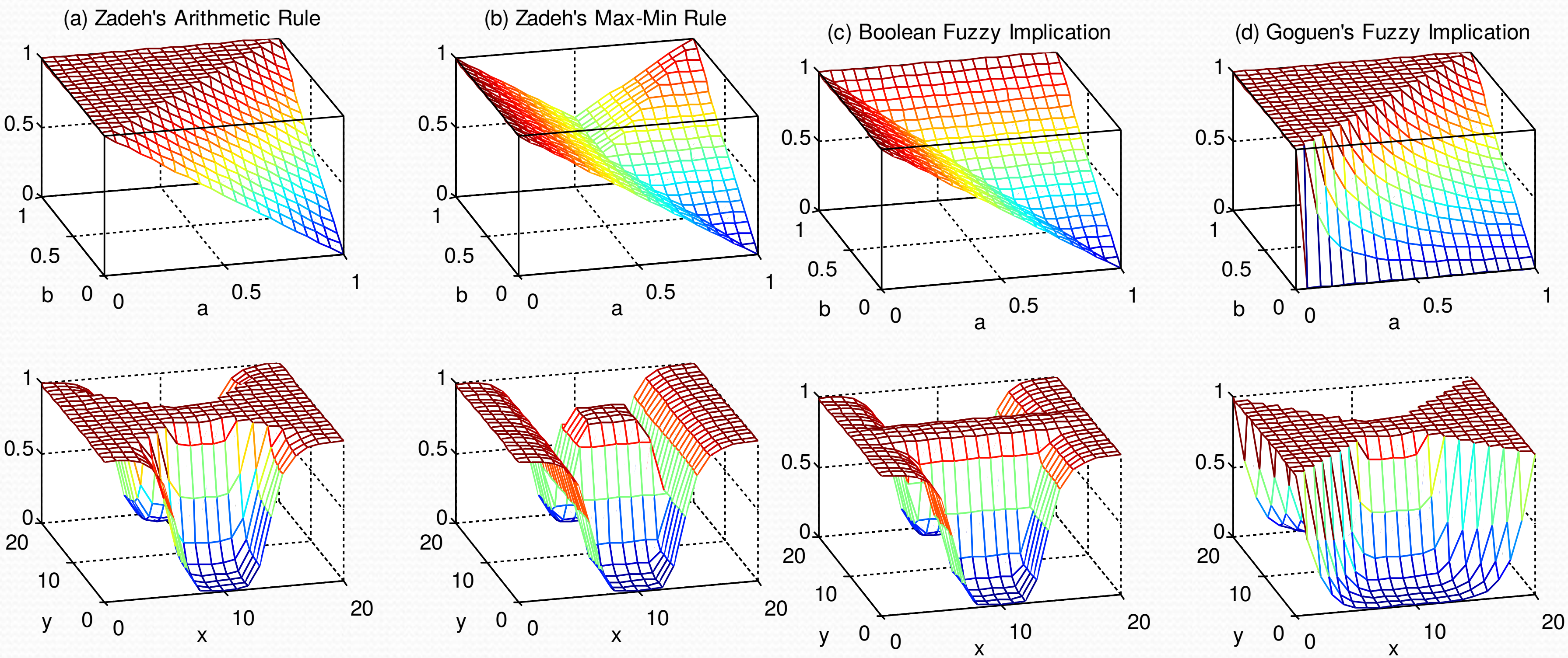
$$m_R(x, y) = max(1 - m_A(x), min(m_A(x), m_B(y)))$$

- Zadeh's arithmetic implication (based on $\neg A \vee B$)

$$m_R(x, y) = min(1 - m_A(x) + m_B(y), 1)$$

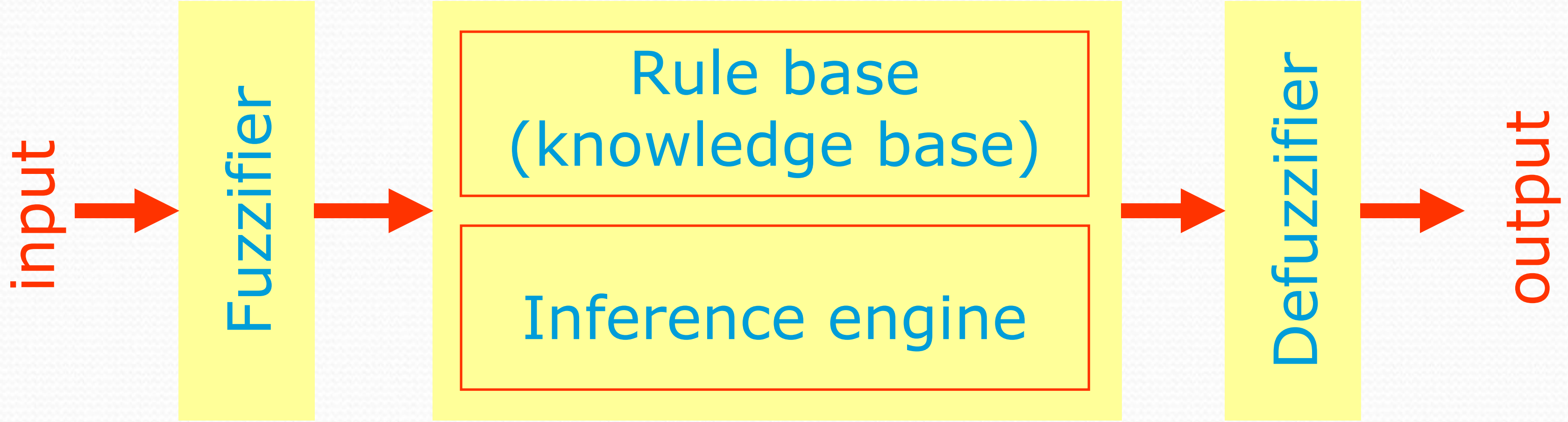
- Goguen's implication

$$m_R(x, y) = min(m_B(x) / m_A(y), 1)$$


(a) Zadeh's Arithmetic Rule
(b) Zadeh's Max-Min Rule
(c) Boolean Fuzzy Implication
(d) Goguen's Fuzzy Implication
1
0.5
0
b
a
y
x
10
20

Building blocks

- Fuzzifier (in the simplest case, turn a measurement into a crisp set)
- Rule base
- Inference engine
- Defuzzifier

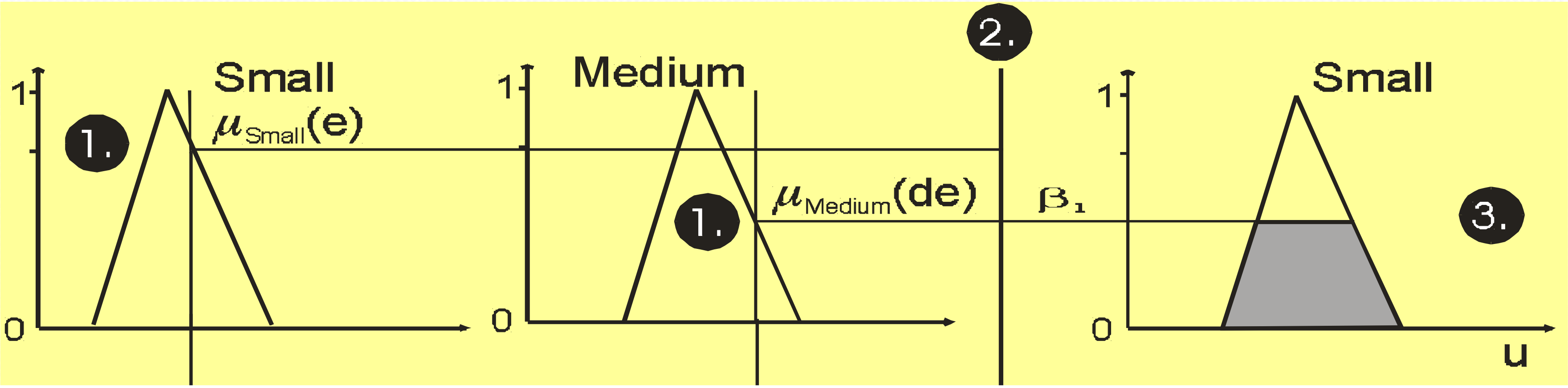


Mamdani Systems: Example 1

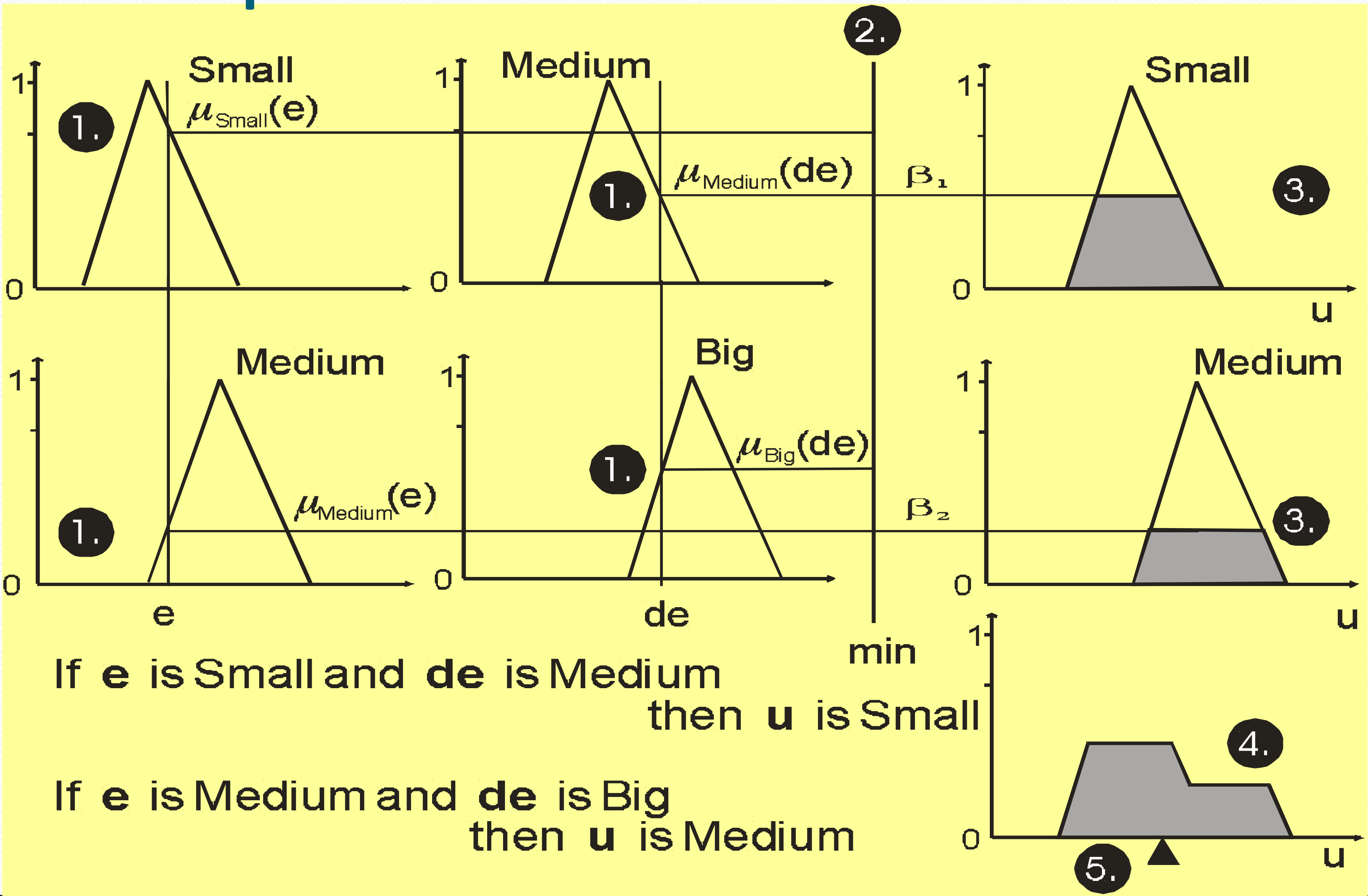
- When given are
  - a fuzzy rule A → B, where A and B are fuzzy sets defined by membership functions $\mu_A(x)$ and $\mu_B(y)$
  - a measurement *a* for A
- The membership function for A → B is defined by
  $\min(\mu_A(x), \mu_B(y))$
- For a measurement *a* the membership for *y* is
  $\min(\mu_A(a), \mu_B(y))$

Mamdani Systems: Example 2

- When rules contain multiple conditions, the min is taken over these conditions



Mamdani Systems: Example 3



Fuzzy observations

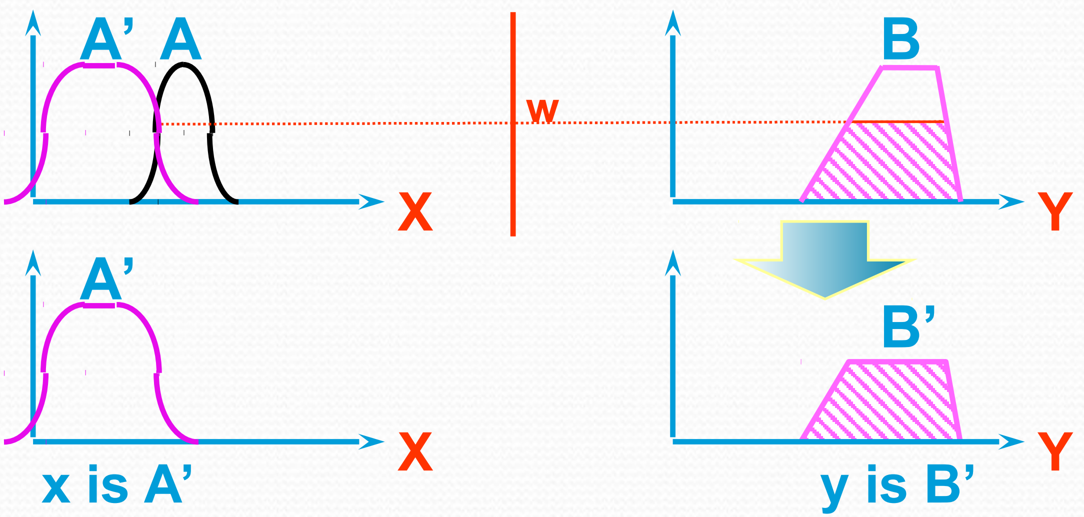
- Rule: if x is A then y is B
- Observation: x is A′ (fuzzy set)
- Conclusion: y is B′ (fuzzy set)

  defined as follows:

$$\mu_{B'}(y) = [\vee_x(\mu_{A'}(x) \wedge \mu_A(x))] \wedge \mu_B(y)$$

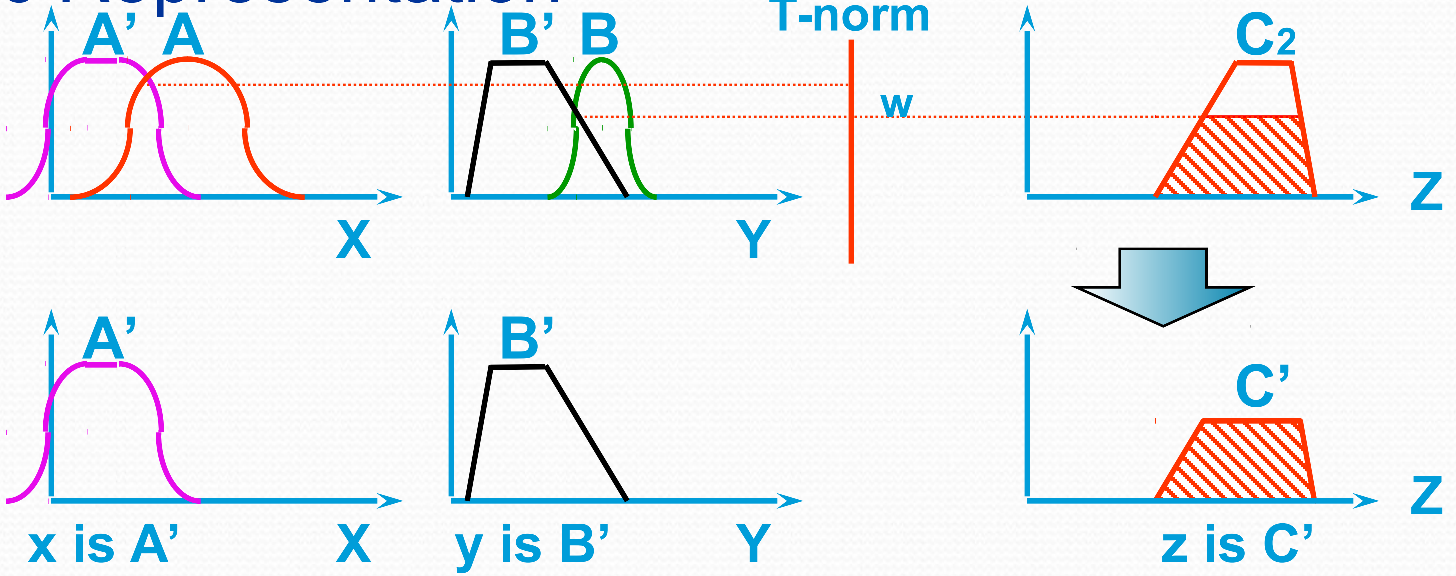
$$= w \wedge \mu_B(y)$$

Graphic Representation

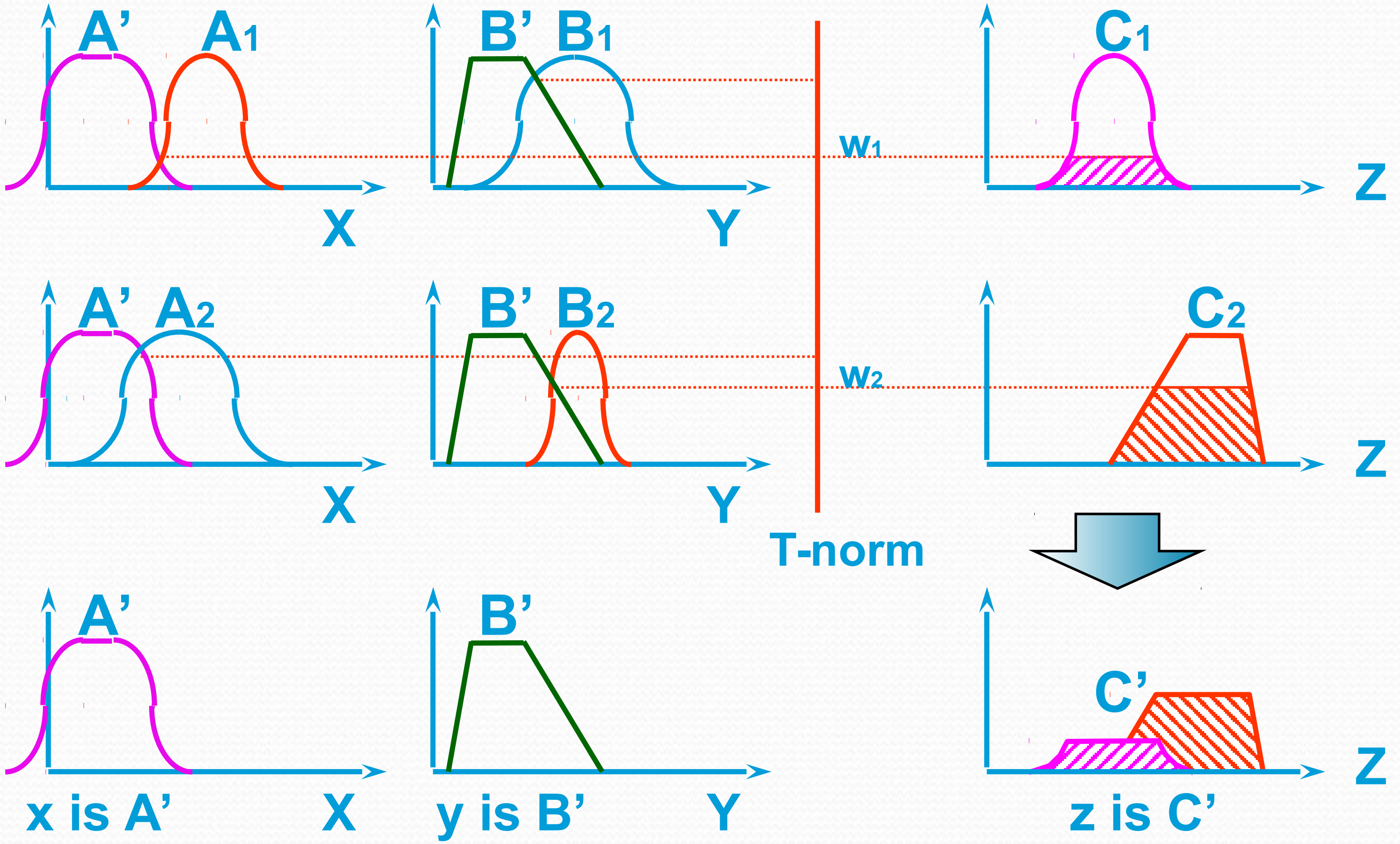


- Rule: if x is A and y is B then z is C
- Fact: x is A′ and y is B′
- Conclusion: z is C′

Graphic Representation



Multiple rules, multiple antecedents

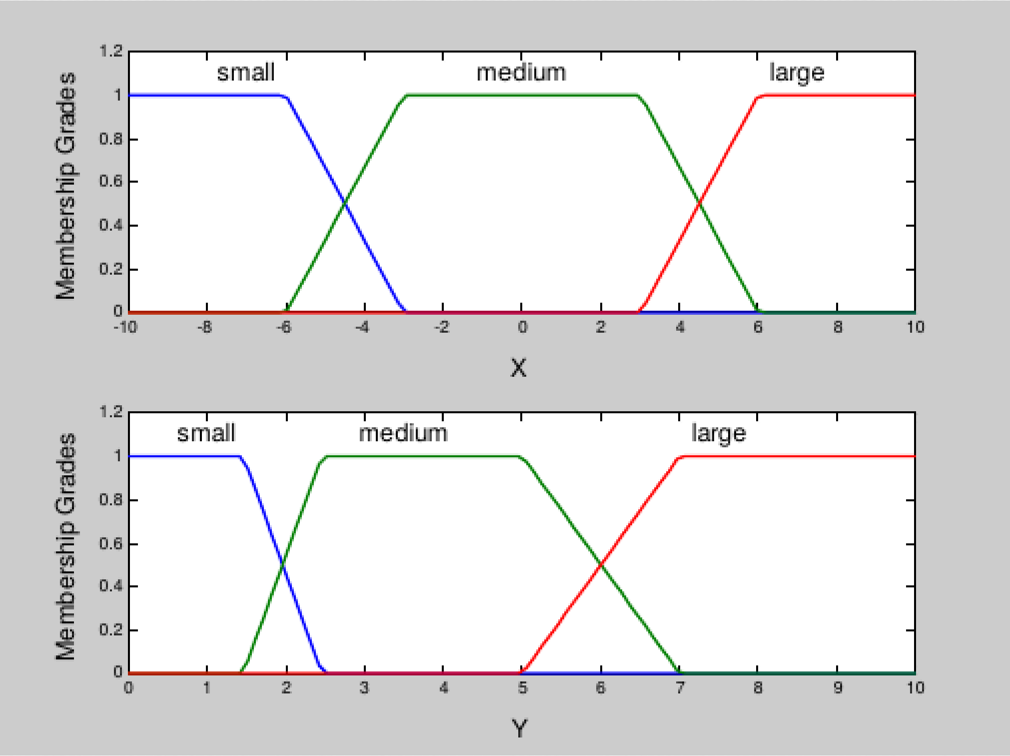


Defuzzification rules

- Centroid-of-area $$z^* = \frac{\int_Z \mu_A(z)\, z\, dz}{\int_Z \mu_A(z)\, dz}$$
- Bisector of area $$\int_{-\infty}^{z^*} \mu_A(z) dz = \int_{z^*}^{\infty} \mu_A(z) dz$$
- Mean of maximum $$z^* = \frac{\int_{Z'} z\, dz}{\int_{Z'} dz}, \quad Z' = \{z \mid \mu_A(z) = \mu^*\}$$
- Smallest of maximum $$\min_{z \in Z'} z$$
- Largest of maximum $$\max_{z \in Z'} z$$

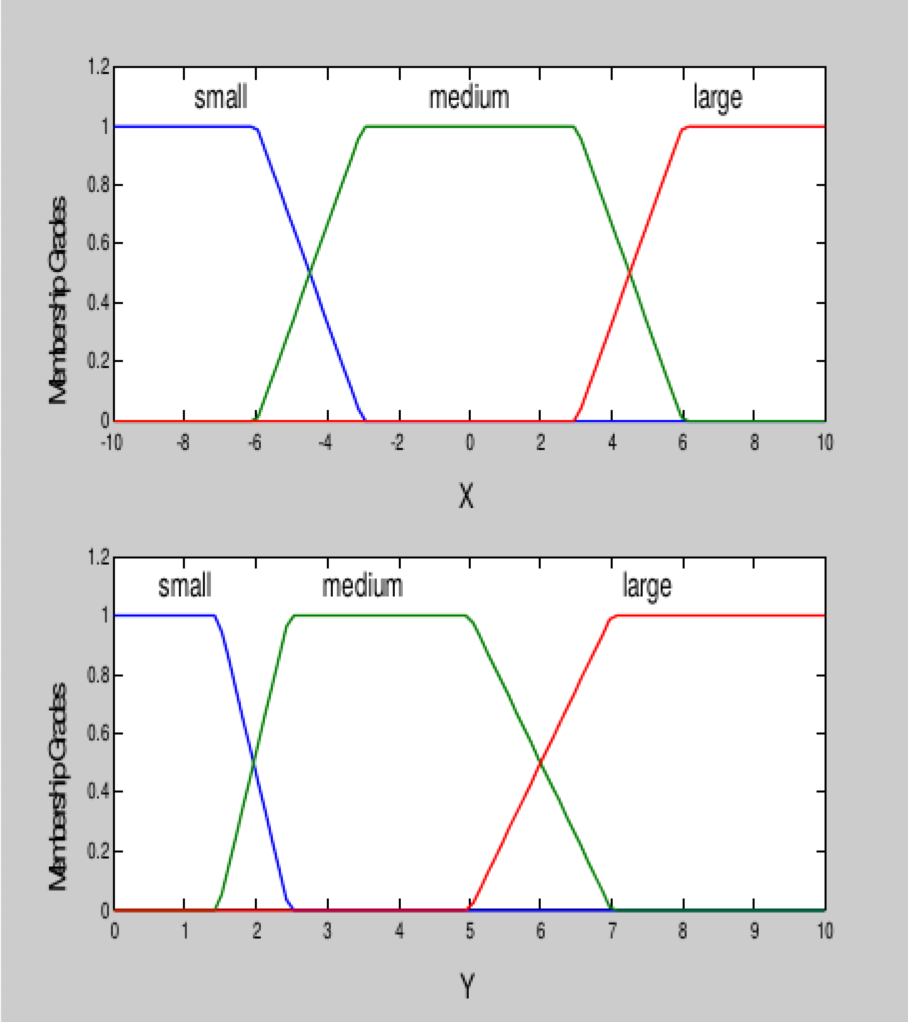
range of values where membership is maximal

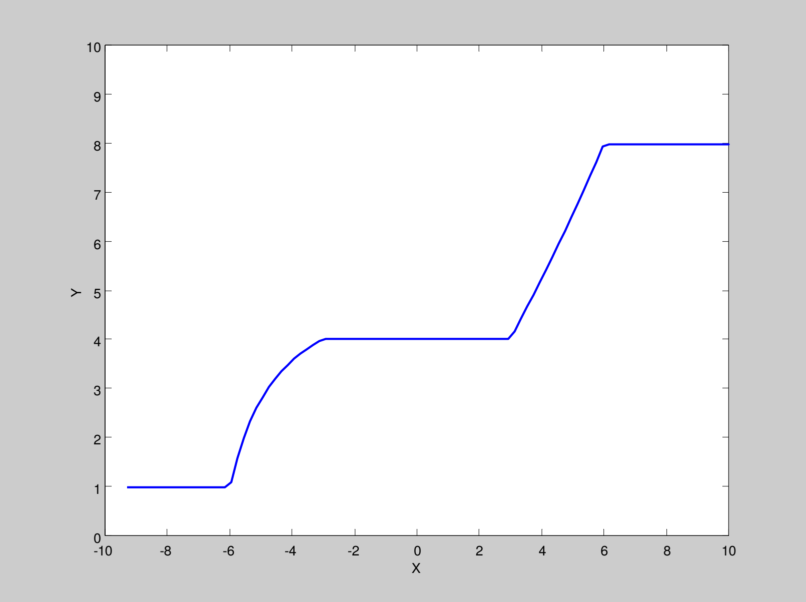
Mamdani - single input



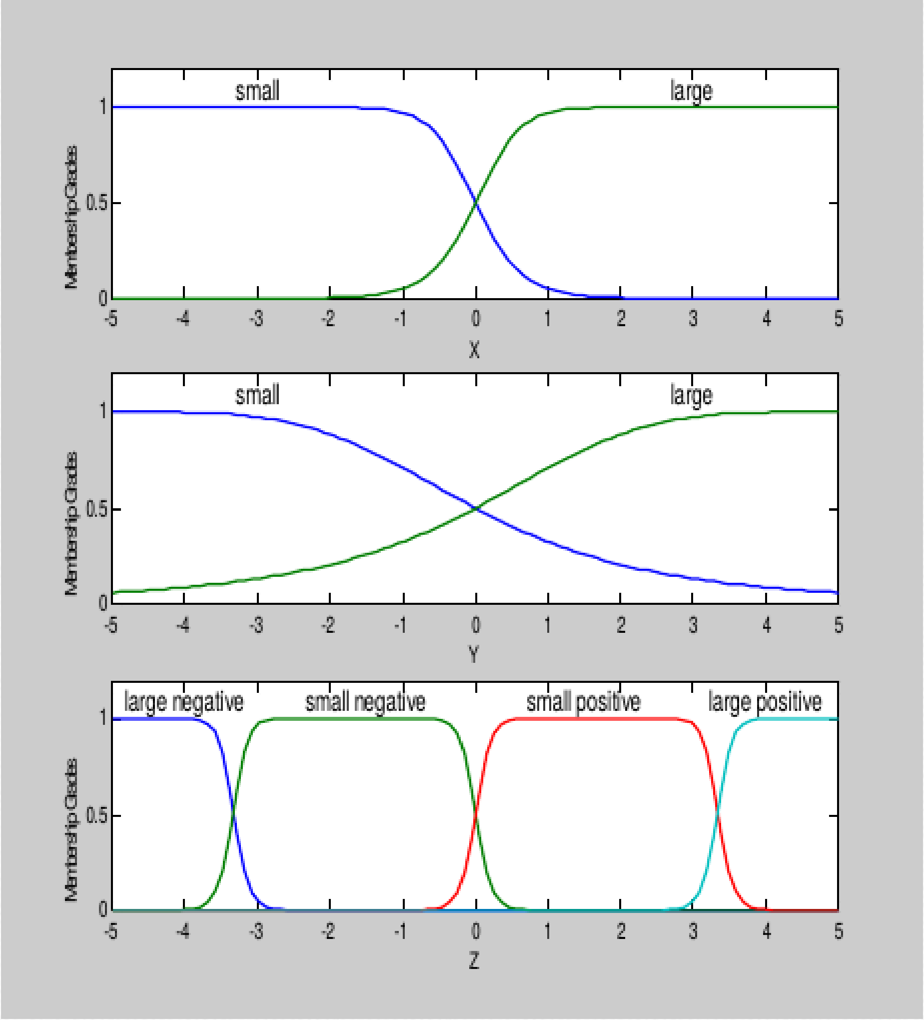
- X is Small → Y is Small
- X is Medium → Y is Medium
- X is Large → Y is Large

Mamdani - single input





Mamdani - double input



- X is Small and Y is Small → Z is negative Large
- X is Small and Y is Large → Z is negative Small
- X is Large and Y is Small → Z is positive Small
- if X is Large and Y is Large → Z is positive Large

Mamdani - double input